

# STUDY ON LONGITUDINAL VELOCITY OF INCOMPRESSIBLE FLUID FLOW OF DUSTY FLUID

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## Abstract

The numerical solution of compressible dusty fluid considering finite volume fraction has been studied. Assuming the velocity and temperature in the jet to differ only slightly from that of the surrounding stream, a perturbation method has been employed to linearize the basic equations. The linearized boundary layer equations have been solved by using double transform technique. Numerical computations have been made to discuss the profiles of velocity of the fluid and particle. It is observed that consideration of finite volume fraction reveals that the magnitude of perturbation velocity of both fluid and particle is reduced significantly

**Index Terms:** Induced Magnetic field, Differential equations, Dusty fluid flow,

## 1. INTRODUCTION:

Compressible jet mixing of a dusty fluid originating from a circular jet has been studied by Dutta and Das [1990], Heat transfer of an axially symmetrical jet mixing compressible dusty fluid, studied by D.K.Dash & Chand Ram [2014]. The Effect of Brownian Motion on the bulk stress in a suspension of spherical particles studied by Batchelor,G.K [1977]. Modelling Dispersion of suspended particulate matter (SPM) in Axi symmetric jet mixing studied by Panda T.C., Mishra S.K., Dash D.K.[2005] and many more has studied jet problem in case of negligible volume fraction of SPM. But this assumption leads to an error which ranges from insignificant to very large. Also this type of assumption is not justified when the fluid density is high or particle mass fraction is large. In the present paper the effect of finite volume fraction in an axially symmetrical jet mixing of a compressible fluid with suspended particulate matter has been studied. Assuming the velocity and temperature in the jet to differ only slightly from that of the surrounding stream, a perturbation method has been employed to linearize the governing differential equations. The resulting linearized equations have been solved by using double Transform technique. Numerical

computation of the integrals giving velocity of both the fluid and particle phase have been made to discuss the results. Consideration of finite volume fraction reveals that the magnitude of perturbation velocity of both fluid and particle is reduced significantly.

## 2. GENERAL GOVERNING EQUATIONS:-

The equation governing the study compressible two-phase boundary layer flow in axi-symmetric case can be written in cylindrical polar coordinates as

$$(1-\phi) \left( u \frac{\partial u}{\partial z} + v \frac{\partial u}{\partial r} \right) = \frac{T}{r} \frac{\partial}{\partial r} \left( \sqrt{Tr} \frac{\partial u}{\partial r} \right) + \frac{\rho_p T}{\tau_m} (u_p - u) \quad (1)$$

$$u_p \frac{\partial u_p}{\partial z} + v_p \frac{\partial u_p}{\partial r} = \frac{u - u_p}{\tau_m} \quad (2)$$

Assuming the flow from a circular, opening under full expansion i.e. no pressure variation occurs throughout the flow, and flow variables of the jet differ only slightly from +0that of the surrounding stream, it is possible to write the velocities, temperatures and particle density in the following form as

$$u = u_0 + u_1, v = v_1, u_p = u_{p_0} + u_{p_1}, v_p = v_{p_1}, T = T_0 + T_1, T_p = T_{p_0} + T_{p_1},$$

$$\rho_p = \rho_{p_0} + \rho_{p_1} \quad \text{where} \quad u_1 \ll u_0, u_{p_1} \ll u_{p_0}, T_1 \ll T_0,$$

$$T_{p_1} \ll T_{p_0}, \rho_{p_1} \ll \rho_{p_0}.$$

The quantities with suffix '0' are the values at the opening and those with suffix '1' are perturbed quantities. Again, the suffix p denotes those variables for the particle. Under the above assumption the governing equation (1) to (2) can be written after dropping the \* and suffix one in the non-dimensional linearized form as follows

$$(1-\phi)u_0 \frac{\partial u}{\partial z} = \frac{(T_0)^{\frac{3}{2}}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \alpha \rho_{p_0} T_0 (u_p - u) \quad (3)$$

$$u_{p_0} \frac{\partial u_p}{\partial z} = (u - u_p) + (u_0 - u_{p_0}) \quad (4)$$

The boundary conditions are

$$u(0,r) = u_p(0,r) = T(0,r) = T_p(0,r) = \rho_p(0,r) = 0 \text{ for } r > 1$$

$$v(0,r) = v_p(0,r) = \frac{\partial u}{\partial r}(z,0) = \frac{\partial T}{\partial r}(z,0) = 0$$

$$u(z,\infty) = u_p(z,\infty) = T(z,\infty) = T_p(z,\infty) = \rho_p(z,\infty) = 0 \quad (5)$$

### 3. METHOD OF SOLUTION:

The governing equations (3) to (4) are linear partial differential equations having seven unknowns  $u, u_p, v, v_p, T, T_p$  and  $\rho_p$ . These equations have been solved by a double transform technique. First employing Henkel transform with respect to the variable r, which is denoted by \* and then Laplace transform with respect to the variable z, which is denoted by

Taking Henkel and Laplace transforms respectively of both sides of the above equations and using the boundary conditions we obtain.

$$\bar{u}^* = \frac{J_1(p)[B u_{p_0} + (C+S)u_{10}]}{PF(S)} \quad (6)$$

$$\bar{u}_p^* = \frac{J_1(p)[C u_{10} + (A+S)u_{p_0}]}{PF(S)} \quad (7)$$

The inversion of (6) to (7) gives

$$u = \int_0^\infty \left[ u_{10} \cosh QZ + \frac{1}{Q} \left\{ \frac{(C-A)}{2} u_{10} + B u_{p_0} \right\} \sinh QZ \right] e^{-\frac{(A+C)Z}{2}} J_0(pr) J_1(p) dp \quad (8)$$

$$u_p = \int_0^\infty \left[ u_{p_0} \cosh QZ + \frac{1}{Q} \left\{ C u_{10} + \frac{(A-C)}{2} u_{p_0} \right\} \sinh QZ \right] e^{-\frac{(A+C)Z}{2}} J_0(pr) J_1(p) dp \quad (9)$$

Where  $F(S) = S^2 + S(A+C) + C(A-B)$ ,

$$Q = \sqrt{BC + \left( \frac{A-C}{2} \right)^2}$$

$$A = \frac{P^2 T_0^{\frac{3}{2}} + \alpha \rho_{p_0} T_0}{(1-\phi)u_0}, B = \frac{\alpha \rho_{p_0} T_0}{(1-\phi)u_0}, C = \frac{1}{u_{p_0}}$$

### 4. DISCUSSION OF RESULT AND CONCLUSION:

In the present paper numerical computation have been made by taking  $P_r = 0.72, u_{10} = u_{p_0} = T_{10} = T_{p_0} = \rho_{p_0} = 0.1, \phi = 0.01$ . The velocity at the exit are taken nearly equal to unity. The improper integrals have been evaluated on using the gauss-three point quadrature rule.

Table 1.1 and table 1.2 shows the profiles of axial fluid perturbation velocity  $u$  and longitudinal perturbation particle velocity  $u_p$  for  $\alpha = 0.1, 0.2, 0.3$  and for different values of Z. It is observed that the axial fluid velocity  $u$  is greater than the axial particle velocity  $u_p$  near the nozzle exit whereas  $u_p$  is greater in the mixing region downstream. Also the magnitude of both fluid and particles are changed as we proceed in the downstream direction. Exit whereas  $u_p$  is greater in the mixing region downstream. Also the magnitude of both fluid and particles are changed as we proceed in the downstream direction.

It is observed that consideration of finite volume fraction reveals that the magnitude of perturbation velocity of both fluid and particle is reduced significantly to that of the result obtained by other researcher where volume fraction of SPM was neglected.

Clearly, it indicates that, consideration of finite volume fraction reveals that the magnitude of perturbation velocity of both fluid and particle is reduced significantly.

**Table1.1: Values of longitudinal perturbation fluid velocity u1**

ALPHA=0.1			ALPHA=0.2			ALPHA=0.3		
Z=0.25	Z=0.50	Z=1.0	Z=0.25	Z=0.50	Z=1.0	Z=0.25	Z=0.50	Z=1.0
1.00E-01	1.02E-01	1.06E-01	1.00E-01	1.02E-01	1.07E-01	1.00E-01	1.02E-01	1.07E-01
1.01E-01	1.02E-01	1.06E-01	1.01E-01	1.02E-01	1.06E-01	1.01E-01	1.02E-01	1.07E-01
1.01E-01	1.01E-01	1.02E-01	1.01E-01	1.01E-01	1.01E-01	1.01E-01	1.01E-01	1.00E-01
9.72E-02	9.09E-02	8.40E-02	9.63E-02	8.95E-02	8.27E-02	9.51E-02	8.78E-02	8.12E-02
4.75E-02	4.71E-02	4.74E-02	4.74E-02	4.70E-02	4.74E-02	4.72E-02	4.68E-02	4.75E-02
2.28E-03	7.64E-03	1.47E-02	3.00E-03	8.76E-03	1.59E-02	3.93E-03	1.01E-02	1.72E-02
-3.03E-04	5.29E-05	2.10E-03	-2.72E-04	2.24E-04	2.73E-03	-2.35E-04	4.93E-04	3.56E-03
-2.31E-04	-2.25E-04	-5.47E-05	-2.28E-04	-2.21E-04	5.53E-05	-2.27E-04	-2.10E-04	2.37E-04
-1.60E-04	-1.84E-04	-1.80E-04	-1.69E-04	-1.84E-04	-1.73E-04	-1.76E-04	-1.84E-04	-1.56E-04
-1.73E-04	-1.85E-04	-1.89E-04	-1.78E-04	-1.85E-04	-1.89E-04	-1.81E-04	-1.85E-04	-1.88E-04

**Table1.2: Values of longitudinal perturbation particle velocity UP1**

ALPHA=0.1			ALPHA=0.2			ALPHA=0.3		
Z=0.25	Z=0.50	Z=1.0	Z=0.25	Z=0.50	Z=1.0	Z=0.25	Z=0.50	Z=1.0
6.62E-01	5.37E-01	3.66E-01	6.62E-01	5.37E-01	3.66E-01	6.62E-01	5.37E-01	3.67E-01
8.19E-01	6.59E-01	4.39E-01	8.19E-01	6.59E-01	4.40E-01	8.19E-01	6.59E-01	4.40E-01
8.32E-01	6.69E-01	4.45E-01	8.32E-01	6.69E-01	4.45E-01	8.32E-01	6.69E-01	4.45E-01
7.89E-01	6.34E-01	4.17E-01	7.89E-01	6.33E-01	4.17E-01	7.88E-01	6.33E-01	4.16E-01
3.84E-01	3.09E-01	2.05E-01	3.84E-01	3.09E-01	2.05E-01	3.84E-01	3.09E-01	2.05E-01
-2.41E-03	-7.20E-04	4.26E-03	-2.35E-03	-4.49E-04	4.90E-03	-2.25E-03	-1.15E-04	5.65E-03
-1.88E-02	-1.46E-02	-8.42E-03	-1.88E-02	-1.46E-02	-8.23E-03	-1.88E-02	-1.46E-02	-7.97E-03
-7.08E-03	-5.55E-03	-3.42E-03	-7.08E-03	-5.55E-03	-3.40E-03	-7.07E-03	-5.54E-03	-3.36E-03
3.49E-03	2.67E-03	1.54E-03	3.49E-03	2.67E-03	1.54E-03	3.49E-03	2.67E-03	1.54E-03
2.86E-03	2.18E-03	1.24E-03	2.85E-03	2.17E-03	1.24E-03	2.85E-03	2.17E-03	1.24E-03

**5. REFERENCE:**

[1] Batchelor, G.K.(1977), published the paper entitled “The Effect of Brownian Motion on the bulk stress in a suspension of spherical particles” in Fluid mech., 83,97-117.

[2] Bender, E.A. (1978), published the paper entitled “An introduction to mathematical modelling”, Wiley, New York.

[3] Buffo, Elwoods and Dyer James S. (1977), published the paper entitled “Model formulation and solutions methods” Wiley Hamilton, New York.

[4] Rath B.K.,Mahapatra,P.K,&Dash D.K,(2017), published the paper entitled “ Effect of Volume fraction along with concentration parameter in the dusty incompressible fluid” in 162Advances and Application in Fluid Mechanics , Volume 20, No 1,2017 ,pp 117-125

[5] D.K.Dash & Chand Ram (2014), published the paper entitled “Heat transfer of an axially symmetrical jet mixing compressible dusty fluid” in International J. of Engg. Research & Indu. Appls.

[6] Panda T.C., Mishra S.K., Das A.K.(1999), published the paper entitled “Modelling the turbulent diffusion of SPM in Ekman boundary layer based on 1<sup>st</sup> order closure” in Acta Ciencia indica, 25m(4), 365-373.

[7] Panda T.C., Mishra S.K., Das A.K.(2001), published the paper entitled “ Modelling of laminar diffusion of SPM in Ekman boundary layer” in AMSE, Vol.70 (5), 47-55.

[8] Panda T.C., Mishra S.K., Das A.K.(2001), published the paper entitled “ Unsteady parallel flow of particulate suspension” in AMSE, Vol 69(1), 1-13.

[9] Panda T.C., Mishra S.K., Panda K.C.,(2002), published the paper entitled “Diffusion of SPM using two phase flow model” in Int.J.for Numerical methods in Fluids, New York.

[10] Panda T.C., Mishra S.K., Dash D.K.(2005), published the paper entitled “Modeling Dispersion of suspended particulate matter (SPM) in Axi symmetric jet mixing” in Far East J. Appl. Math. 20(3) (2005), 289 – 304.

[11] Rath, B.K., Behera, G.K., and Dash, D.K. (2015). Published the paper entitled “Solution of Longitudinal velocity of the fluid and the particle of the dusty fluid with the effect of volume fraction in the incompressible fluid of SPM” in Adv. Appl. Fluid. Mech. 18, 155-162

[12] Dash D.K. & Rath B.K.,(2009) published the paper entitled” Effect of volume fraction in axi-symmetric jet mixing of incompressible flow In cylindrical polar co-

ordinates” in Far east journal of applied mathematics, Vol 34, No. 1,2009, Pages 83-94.

[13] Dash D.K. & Rath B.K.,(2008) published the paper entitled “Modelling of boundary layer equation in axisymmetric, incompressible Flow In cylindrical polar coordinates and its simplification” in Impact, J.sci.Tech Volume2(1) 2008,FIJI ISLAND, page 1-8.

[14] Dash, D.K; Rath, B.K., published a paper entitled “Effect of volume fraction on temperature in axi symmetric jet mixing of incompressible flow in cylindrical polar coordinates “ in Applied Science Periodical in volume xiii, No -1 , February 2011

